Modelling Conflict and Cooperation: A Mathematical Approach to International Relations

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***Muhammad Imran Chaudhry[1]**

Abstract
Contemporary research in international relations necessitates innovative and interdisciplinary approaches that not only deepen our understanding of the complexities of the global system but also contribute to practical solutions for conflict resolution and the advancement of peace and stability. One of these integrated approaches is mathematical modelling, which includes game theory, network analysis, and dynamical system models. The game theory model offers a framework to analyse situations involving mutually dependent decision-making and the impact of these decisions on others. Likewise, network analysis and dynamical system models can be utilised to comprehend various scenarios of conflict and cooperation between states. This paper presents a hypothetical formulation of a game-theoretical model to assess the applicability of mathematical modelling to real-world situations. It discusses the use of utility functions and analyses Nash equilibrium, Pareto optimality, and the security dilemma to grasp the complexities of decision-making. The research focuses on prominent theoretical frameworks of international relations, namely realism and idealism, to demonstrate that mathematical modelling is equally effective in studying the dynamics and behaviour of an international system where sovereign states make strategic choices based on their national interests.

Keywords
Mathematical modelling, conflict resolution, game theory, geopolitical analysis, international cooperation

Introduction
The complex interplay among states in international relations has sparked longstanding scholarly interest and inquiry. Over the years, various approaches have been considered to comprehend the multifaceted dynamics underpinning statecraft, diplomatic ties,

[1] *Corresponding Author: *Muhammad Imran Chaudhry* is a visiting scholar at the University of California, Berkeley, USA.
E-mail: emmichaudhri@outlook.com

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threat perception, conflict resolution, trade, and connectivity. On the one hand, these endeavours inspire optimism about the possibilities of progress in the field of international relations; on the other hand, many have highlighted numerous obstacles to achieving a sustainable conclusion. The real challenge is deciding whom to regard as authoritative since knowledge in this field is not as cumulative as it is in the study of natural sciences.

From the Greeks to the present day, fundamental questions concerning war and peace (in other words, conflict and cooperation) have persisted, and if they have changed, that variation is too slight to discern. This is because theorists often find themselves moving in cycles, referring to classical ideas as each peer group engages with similar fundamental inquiries influenced by their worldviews (shaped by circumstances), which results in repetitive analyses while progress remains elusive.

Today, several approaches, including realism, idealism, structuralism, conservatism, pluralism, and radicalism, lay the foundation for theories of international relations. However, nearly all these ideologies encounter challenges, as they are shaped by contextual worldviews, limiting their ability to isolate variables and produce generalisable insights. For instance, realist theories assume that conflict is inevitable, while idealist frameworks prioritise cooperative institutions. Neither tradition systematically quantifies decision-making processes or strategic interactions. This has led to repetitive, non-cumulative scholarship and reliance on subjective interpretations of historical events.

In international relations, a theory starts by identifying the factors responsible for a conflict and then proposes a hypothesis or explanatory model as a remedy. Consequently, it is not easy to deconstruct and reconstruct a phenomenon by removing it from its context. A state can be peaceful or aggressive, and these characteristics are contextual to the situation. At the same time, qualitative analysis in international relations faces limitations arising from both contextual and non-contextual elements of the definitional process unless a broad consensus on relevant terms or available variables is established. As a result, it is uncommon to assess the effectiveness of a theory solely on scientific grounds.

The complexity of the field also stems from competing methodologies: qualitative approaches encounter challenges related to standardisation, whereas quantitative methods tend to oversimplify political dynamics. For instance, the billiard-ball model (balance of power) and the cobweb approach (interdependence) underscore systemic interactions but lack the tools to predict behavioural shifts. Similarly, debates about anarchic versus hierarchic systems, although philosophically rich, fail to operationalise variables such as national interest or threat perception into testable hypotheses. To tackle this issue, an integrated approach is increasingly necessary to address the gaps left by definitional ambiguities and to seek consensus on specified variables.

In this regard, the integration of mathematical principles into international relations holds significant potential for distilling the complexities of diplomatic strategies into quantifiable, predictable models. Mathematical modelling offers a novel pathway to analyse strategic choices objectively, such as the Nash Equilibrium in a security dilemma, identify patterns in state behaviour through network analysis, and simulate scenarios like trade disputes and alliance formations to test theoretical assumptions. This paper, therefore, aims to contribute to the field of international relations by examining the capacity of mathematical modelling to elucidate the enigmas of conflict and cooperation. Through the application of game theory, network analysis, and dynamic systems, the complex network of international relations can be scrutinised, endeavouring to reveal concealed patterns and structures influencing national behaviour. The paper transitions from mathematical approaches to the real-world landscape, seeking to bridge the gap between both disciplines and provide insights for future research.

Theocratical Framework

International relations scholarship is grounded in two dominant theoretical paradigms—realism and liberalism—which offer contextual understanding and insights into the complex dynamics of international conflicts and cooperation. Drawing on these theories, this paper attempts to integrate mathematical modelling to analyse real-world situations and aids in defining assumptions and parameters of anticipated models, suggesting effective conflict resolution strategies. For instance, realism offers views on power dynamics and state behaviour that may stimulate mathematical models of strategic interaction for a tangible conclusion (Waltz, 1979; Mearsheimer, 2001), such as game-theoretic frameworks that suggest cooperative behaviour, which is more likely to emerge when states repeatedly interact and share interests (von Neumann & Morgenstern, 2007), thus simulate conflict escalation or bargaining scenarios. Similarly, liberal theories emphasise interdependence and the role of international institutions and cooperation, offering valuable insights for modelling cooperative behaviour and shared interests (Kant, 1795; Keohane & Nye, 1977). It underpins network analysis models demonstrating how states significantly affect economic outcomes when examining centrality measures in international trade networks (Wasserman, 1994; Jackson, 2008; Jackson, 2010). Such models can map economic or diplomatic ties among states.

To address gaps in these paradigms, other theoretical perspectives, such as constructivism, focusing on norms, identities, and belief systems (Wendt, 1992; Cox, 1981) and institutionalism, examining decision-making processes and organisational roles (Keohane, 2005; Pierson, 1996), offer complementary insights. For instance, constructivist principles can enrich dynamical systems models by capturing how normative shifts alter state behaviour over time, while institutionalist logic can refine agent-based models to simulate collective action challenges within international organisations. This interdisciplinary synthesis, bridging political theory, computational social sciences, and mathematical modelling, enables assumptions for model design, e.g., defining rational actors in game theory vs. socially constructed agents in dynamical systems, and strengthens interpretive rigour when evaluating outcomes. Dynamical system models demonstrate that slight changes in initial situations lead to considerably different outcomes, stressing early diplomatic interventions or engagements (Hirsch et al., 1974; Strogatz, 2024).

Integrating mathematical modelling into policymaking can lead to effective and sustainable conflict resolution strategies. By formalising theoretical assumptions into testable models, e.g., realist 'power-maximising' agents versus constructivist 'norm-sensitive' agents, this approach clarifies which paradigms best explain observed behaviours. For policymakers, dynamical systems models highlight the urgency of early mediation in escalating conflicts, while network analysis identifies pivotal states capable of brokering cooperation. Explicitly linking theories to methodologies not only enhances reproducibility but also grounds abstract models in politically meaningful contexts. It is, therefore, essential to develop workable models that reflect the multifaceted nature of global politics and pave the way for effective policymaking. The paper, while reflecting on realist and idealist approaches to conflict resolution, attempts to address several key questions, including the application of game theory in modelling conflict and cooperation scenarios, how network analysis can provide insight into states' behaviours, how to use dynamical systems models to comprehend states' relationships, and what the advantages are of employing mathematical modelling in international relations. These questions are addressed by employing mixed-methods research that integrates theoretical analysis with computational and mathematical modelling to explore the dynamics of conflict and cooperation. It utilises secondary sources and implements analytical techniques such as equilibrium analysis to identify stable outcomes of cooperation or conflict, network analysis to reveal coalition patterns, and dynamic systems analysis to assess how pre-conflict trust levels influence long-term trajectories.

International Relations: Understanding of Conflict and Cooperation

Conflict and Cooperation are two different but interconnected expressions in international relations. Cooperation is when actors adjust their behaviour to actual or anticipated preferences of others through a process of policy coordination (Keohane, 2005), aiming to yield rewards for all parties involved, though not necessarily in equal measure. Conversely, conflict or competition is goal-seeking behaviour that aims to reduce the gains available to others or to impede their want-satisfaction (Milner, 1992). Both have a significant bearing on international relations, as the notion of a modern nation-state was born out of conflict (Thirty Years' War) and led to international cooperation (Peace of Westphalia). Despite a long journey of three centuries, the most fundamental inquiries in international relations involve the root causes of interstate conflicts, methods to prevent or mitigate conflicts, and pathways leading to increased global cooperation.

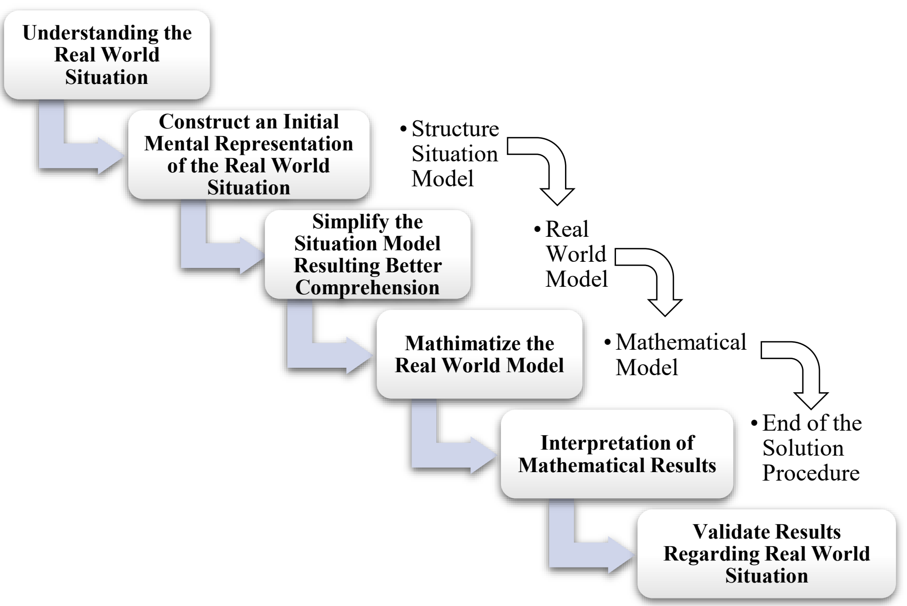
Today, when addressing these inquiries, the prevailing paradigm in international relations, political realism, underscores the persistent inclination for conflict among self-interested states seeking security in an anarchic world. This environment lacks a central authority to safeguard weaker states from the powerful or guarantee their security. In turn, states seek to protect their interests by asserting control, bolstering their military capacities, and forming alliances, thus generating a security dilemma (Cerny, 2000). Consequently, realists perceive international cooperation to be infrequent, transitory, and delicate. Although in many instances, states assisted each other, especially in natural calamities and disasters, and established institutions for coexistence and global cooperation, it is still constrained by enforcement issues and each state's inclination for relative gains in any potential agreement due to its systemic vulnerability (Morgenthau, 1949; Waltz, 1979).

At its most extreme, states find themselves in a security condition characterised by mutual distrust resembling a prisoner's dilemma (Halkos et al., 2021). The equilibrium in the international system still relies on a balance of power or limited cooperation. This dynamic represents the highest attainable goal. War is viewed as a natural aspect and often described as 'diplomacy by other means,' a concept that underscores the role of war as a tool for achieving diplomatic objectives without peaceful negotiations (Clausewitz, 1989). It is important to note that international relations do not perpetually exist in a state of war; instead, they operate within the shadow of war as the ultimate arbitrator. Liberalists posit that the inevitability of war can be challenged. They emphasise interdependence and the potential for increased cooperation within the context of reduced anarchical conditions. This emphasis on the potential for increased cooperation offers a hopeful perspective on the future of international relations (Jervis, 1999). In short, realism and liberalism are at play to seek dominance in their respective cognitive domain, translating into a chaotic disposition of world politics.

Mathematical Modelling and International Relations

Recent scholarly trends suggest a significant shift from traditional political ideologies within the international system. The strategy of containment, once aimed at protecting established political norms from communist influence, is increasingly viewed as outdated in light of a rapidly evolving global landscape shaped by advancements in automation, communication, and artificial intelligence. These changes transform the socio-political environment and affect wealth distribution and individual self-esteem. As transnational economic and financial networks become more prominent, strategies for achieving goals are moving from coercive to cooperative approaches. This shift underscores a growing scepticism toward conventional leadership styles and political decision-making, with an emphasis on negotiation and strategic thinking that prioritises long-term consequences over short-term gains. The challenges associated with realism are now being addressed within the framework of neoliberalism or through interdisciplinary approaches. In this context, one of the integrated (interdisciplinary) approaches is the mathematical modelling of conflict and cooperation. It involves translating our conceptual understanding of real-world phenomena into mathematical language to solve specific problems. It is a cyclical process, transitioning from the real world to the mathematical realm and returning to reality as demonstrated in the figure below (Krawitz et al., 2022).

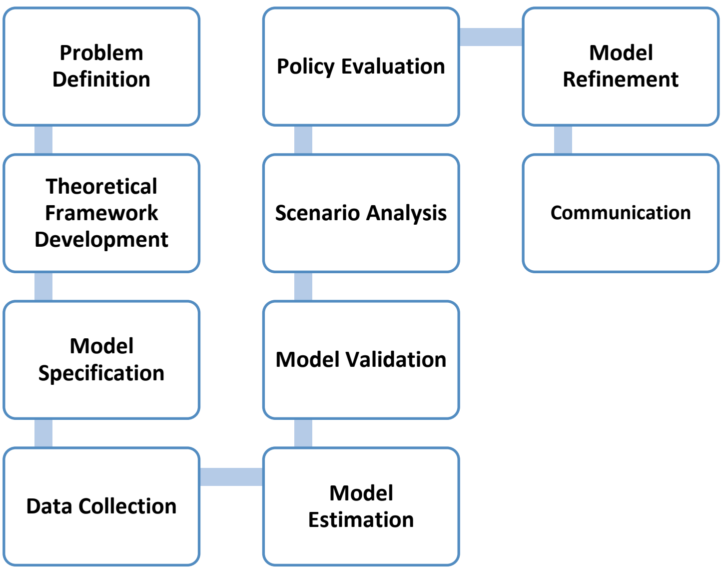
Figure 1. Mathematical Modelling to Solve a Real-World Problem



Source: Author

Mathematical modelling presents a systematic process that consists of several vital steps, as shown in the figure below. Starting from identifying and defining a real-world problem (or devising a research question), it leads to developing a theoretical framework, identifying corresponding variables (dependent and independent), and outlining their relationships. It provides a foundation for formalising an appropriate model using differential equations and statistical tools. For model inclusion, relevant data (qualitative and quantitative) is collected, followed by assessing the defined parameters of the model, inserting the data and evaluating the model's performance and validation. The model is then employed to simulate different scenarios for estimation to assess the effectiveness of various policy options by analysing results. The model's credibility rests on its refinement through new data and the insertion of emerging trends. The concluding step is communicating findings and insights using clear and accessible language to all stakeholders. This systematic approach can promote advanced theoretical understanding and help analyse complex international relations issues.

Figure 2. Process of Mathematical Modelling in International Relations



Source: Author

Taking account of realism and liberalism while focusing on security and conflict, mathematical modelling has a significant role. Various theoretical frameworks of international relations can be interpreted into mathematical models for practical analysis, such as game theory, which applies to identifying a prisoner's dilemma or security dilemma, and dynamic systems theory, which corresponds to understanding the arms race phenomenon. Realists often assume rational actors and focus on relative gains, which can be modelled using the mathematical concepts of utility maximisation and Nash equilibrium (CFI Team, 2024; Eldridge, 2024). In such models, military capability, economic power, and geopolitical position act as variables that help to analyse the dynamics of conflict and cooperation. With regard to liberals' views, emphasising cooperation, interdependence, and international institutions can be modelled by using game theory (relevant to institutional design and cooperative games), or statistical models can be used to analyse the impact of trade on conflict. These models often assume actors with diverse preferences and a focus on absolute gains. Variables such as democracy, economic interdependence, and institutions are incorporated into liberal models to analyse the dynamics of cooperation and peace.

In applying mathematical modelling to realist and liberalist approaches, researchers formalise various models to test their hypotheses, examine different assumptions and scenarios, evaluate policy interventions, and provide a systematic understanding of the phenomenon under scrutiny. In order to comprehend this process, a hypothetical model combining elements of realism and liberalism (conflict and cooperation between two states) is given below.

Table 1. Model of Conflict and Cooperation

<table>
<tr><td>Assumptions</td><td>• Two sovereign states, State A and State B, operate in an anarchic international system.
• Both possess military capabilities (M) and economic resources (E).
• They have conflicting interests. Each state has the option to either cooperate (C) or conflict (K).
• Resultant payoffs correspond to their security (S), economic benefits (B) and reputation (R).</td></tr>
<tr><td>Game Structure</td><td>• In a simultaneous move game, states can choose between two strategies: C and K. The combination of the selected strategies determines the payoffs resulting from States’ choices.</td></tr>
<tr><td>Payoff Matrix</td><td>|	A(C)	A(K)
B(C)	S=3, B=2, R=1	S=1, B=4, R=1
B(K)	S=4, B=1, R=-1	S=2, B=3, R=0 |</td></tr>
<tr><td>Description of Matrix</td><td>It is a simple model structured to depict the payoffs for State A in each possible combination of strategies, where State B's strategies are implied; however, it can be structured both ways.

In the given Matrix, rows represent the strategies of State A, and columns represent State B’s strategies in case of conflict (K) or cooperation (C). The entries in the matrix correspond to payoffs denoted as S, B, and R (the first value for State A’s payoff and the second value for State B’s payoff).

Numbers are the relative magnitude of payoffs referred to as Utility Value. These values (1=Lowest, 2=Low, 3=Medium, and 4=Highest) for State A and State B are identified. In terms of security e.g., 1=Border Vulnerability, 2=Minor Skirmish, 3=Deterrence, and 4=Robust Defence System), economic benefits (1=Low Investment and Minimal Trade, 2=Moderate Trade with Some Investment, 3=Substantial Investment and Significant Trade, and 4=Huge Investment and Major Economic Partnership) and reputation (0=Neutral, -1=Diplomatic Isolation and 1=International Praise with Diplomatic Recognition).</td></tr>
<tr><td>Model Equations</td><td>• State A’s Utility Function: $U_A = S_A + \beta * E_A + \gamma * R_A$ (reflecting its preferences over security, economic welfare, and reputation).
• State B’s Utility Function: $U_B = S_B + \beta * E_B + \gamma * R_B$ (reflecting its preferences).
• Dynamics: $dM_A/dt = \alpha * (M_B - M_A) * K_A - \delta * C_A$ (capturing the impact of conflict and cooperation on military capabilities).
• Dynamics: $dE_A/dt = \varepsilon * (E_B - E_A) * C_A - \zeta * K_A$ (capturing the impact on economic resources).
• Dynamics: $dR_A/dt = \eta * (C_A - K_A)$ (capturing the impact on State’s reputation).</td></tr>
</table>

Explanation of Equations	The model is based on utility function and dynamics to capture the preferences of both States and evolutions in terms of their military capabilities, economic benefits and international reputation. The utility function **U_A = S_A + β * E_A + γ * R_A** represents State A's preferences considering S_A is State A's security (military strength, defence capabilities, or territorial control), E_A is State A's economic welfare (GDP, trade, or resource acquisition), R_A is State A's reputation (international prestige or diplomatic influence), whereas β and γ are weights representing relative importance of economic benefits and international reputation in State's overall utility. The same is true for U_B = S_B + β * E_B + γ * R_B representing State B's policy-options. The dynamics equation **dM_A/dt = α * (M_B - M_A) * K_A - δ * C_A** describes how State A's military capabilities (M_A) change over time because of conflict (K_A) and cooperation (C_A) with State B. In this equation, dM_A/dt shows the rate of change of State A's military capabilities over time (t). The difference in military capabilities between State A and State B is M_B - M_A. Whereas K_A shows conflict level of State A and C_A provides cooperation level of State A. α and δ are effectiveness of military actions and impact of cooperation on military capabilities. Second and third equations can also be interpreted likewise.
Variables	• M_A and M_B denote Military Capabilities (e.g., Troops and Weapons). • E_A and E_B denote Economic Resources (e.g., GDP and Trade). • C_A and C_B are Cooperation Levels ($0 \leq C \leq 1$, e.g., Diplomatic Efforts and Joint Projects). • K_A and K_B are Conflict Levels ($0 \leq K \leq 1$, e.g., Military Actions and Sanctions). • β and γ are parameters representing the relative importance of economic welfare and international reputation. • α, δ, ε, ζ, and η represent parameters regarding the effectiveness of military and economic actions and the impact of conflict and cooperation on States' reputation.
Clarification of Variables:	Variables (Military Capability, Economic Resources, Cooperation Level and Conflict Level) and parameters (relative importance of economic welfare (β), international reputation (γ), effectiveness of military and economic actions (α, δ, ε, ζ) and impact of conflict and cooperation on states' reputation (η)), used in the model clarify the meaning and context to understand the model and its dynamics. The dependent variables are State A's military capabilities, economic resources and reputation because their values change over time based on dynamic equations. The are independent variables, such as State A's cooperation and conflict levels and State B's military capabilities, economic resources, and cooperation and conflict levels. Their independence depends upon the influence they exert on the changes that occur among dependent variables.
Analysis Methods	• Nash Equilibrium Analysis (to determine optimal strategies). • Stability Analysis (to determine the behaviour of the system). • Sensitivity Analysis (to examine the impact of changes in parameters).
Extended Analysis	**Nash Equilibrium Analysis:** It aims to find the optimal strategies for players in a game where no player can improve payoff by unilaterally changing strategy (Nash, 1950). In the given model, it is to find the values of C_A, C_B, K_A, and K_B by using partial derivatives that satisfy the equation ∂U_A/∂C_A = 0, ∂U_A/∂K_A = 0, ∂U_B/∂C_B = 0, ∂U_B/∂K_B = 0, representing the optimal strategies (subject to the constraints 0 ≤ C_A, C_B, K_A, K_B ≤ 1, where 0 represents no conflict or cooperation and 1 denotes full cooperation or maximum conflict). For example, ∂U_A/∂C_A = 0 means the utility of State A remains unchanged if it changes its cooperation level (C_A) slightly, assuming that the strategy of State B is

	constant. The same is the case of $\partial U_A/\partial K_A = 0$, where the conflict level (K_A) is slightly changed. In the case of $\partial U_B/\partial C_B = 0$ and $\partial U_B/\partial K_B = 0$, the utility of State B does not change, cooperation (C_B) and conflict levels (K_B) are slightly changed, but the strategy of State A remains constant. It shows that no State, either A or B, can improve its utility by changing its conflict or cooperation level unilaterally when the strategy of other State remains constant. **Stability Analysis:** It analyses the behaviour of a system, whether it maintains or restores its equilibrium when acted upon by forces tending to displace it (Nagurney & Zhang, 1996, p.15). Stability analysis is carried out using different methods. The model, the behaviour of the system around the equilibrium point ($C_A^*, C_B^*, K_A^*, K_B^*$) can be analysed by Jacobian matrix of partial derivatives, such as $J=\partial(dM_A/dt, dE_A/dt, dR_A/dt, dM_B/dt, dE_B/dt, dR_B/dt)/\partial(C_A, C_B, K_A, K_B)$. It is used to measure how changes that take place in variables (cooperation and conflict levels) affect the rate of change in other variables (military capabilities, economic resources and international reputation)
	Sensitivity Analysis: It analyses how changes in parameters affect the behaviour of a system or how different values of independent variables affect dependent variables. It is used under certain conditions (Vipond, 2024). It involves partial derivatives of utility functions and dynamics equations with respect to parameters ($\beta, \gamma, \alpha, \delta, \varepsilon, \ldots$): $\partial U_A/\partial\beta$, $\partial U_A/\partial\gamma$, $\partial U_A/\partial\delta$, $\partial U_B/\partial\beta$, $\partial U_B/\partial\gamma$, $\partial U_B/\partial\delta$, $(dM_A/dt)/\partial\alpha$, $(dM_A/dt)/\partial\delta$, … For example, $\partial U_A/\partial\beta$ measures change in the Utility of State A (U_A) with respect to a small change in parameter β, and $(dM_A/dt)/\partial\delta$ measures change in the rate of change of State A's military capabilities (dM_A/dt) regarding a small change in parameter δ. Sensitivity analysis helps to understand how the system responds to changes in the parameters and results in optimal policy decisions and enhances system resilience

Source: Author

The model above (Payoff Matrix) incorporates descriptive variables and mathematical equations to study the dynamics of conflict and cooperation between states (A and B). In the given scenario, the strategic interaction between both states in a simultaneous move is known as a normal-form game in game theory. At the same time, it combines elements of realism, emphasis on military capabilities and security, and liberalism, focus on cooperation and economic benefits, to analyse the conditions under which cooperation emerges, the impact of different strategies on security, economic, and reputational outcomes. These outcomes are based on the choices adopted by both states, capturing trade-offs between security, economic benefits and reputation. The model highlights the complexity of decision-making in anarchic international systems.

From a realist's viewpoint, states may choose conflict as a rational approach to pursue their interests, even if it yields suboptimal outcomes. The pursuit of security by one state can lead to insecurity for another state, showing the security dilemma. If State A perceives that State B will act against it, even then, it is rational for State A to choose conflict to guarantee its security. Mearsheimer contends that pursuing security primarily drives states and will engage in conflict when necessary to achieve it (Mearsheimer, 2014). Therefore, the existence of a security dilemma is validated. Now, if State A increases its military capability, that would also lead to State B's perception of insecurity and prompt State B to enhance its military capabilities. While pondering the security dilemma, Jervis explains how states fortifying their security can sensitise insecurity for others (Jervis, 1978). If State A has military superiority, it can dissuade

State B from opting for the option of conflict. Waltz maintains that military capability is critical to states' behaviour and subsequent outcomes (Waltz, 1979).

According to liberalists, cooperation is possible among states in an anarchic system; however, it is often challenging due to the risk of defection and thus needs credible commitments. Oye observes that there are hitches to attaining cooperation in anarchic systems, which necessitate credible commitments (Oye, 1986). Although the model illustrates the same, even when states share common interests, it emphasises institutions' role in facilitating cooperation among states and reducing the risk of conflict. A similar view was put forward by Keohane, saying that international cooperation is possible even in the absence of a central authority, but it is because of the development of international institutions (Keohane, 2005). The model further explains how cooperation levels and states' reputation influence their behaviour and resultant outcomes, similar to Krasner, who examines the role of international institutions in shaping states' behaviour and promoting cooperation (Krasner, 1983). The model also verifies the liberalist assumption of interdependence and explains how economic interdependence incentivises cooperation among states seeking to avoid conflict. It is a key factor in promoting peace and stability (Russett & Oneal, 2001). Additionally, the model emphasises the importance of states' reputation in maintaining cooperative behaviour and credibility. Mercer argues that states are motivated to maintain a reputation for credibility and reliability (Mercer, 1996).

Game Theory and Conflict and Cooperation

Game theory is an effective instrument of applied mathematics used in modelling across various disciplines to analyse how decisions are being made and their impact on others. It is an analytical tool to examine a situation in which mutually dependent decision-making occurs that necessitates each participant (player) to take others' strategies into account while making decisions, even if they have similar, opposing, or divergent interests (Davis & Brams, 2024). The application of game theory in international relations has already been demonstrated in the abovementioned model, which helps to understand the decision-making in a complex system and determines the resulting outcome. However, the preferences and beliefs of states involved in the situation significantly shape the outcomes. There are approaches like Prisoner's Dilemma, Chicken Game, and Stag Hunt that help to investigate the situation thoroughly in one way or the other. For example, a prisoner's dilemma occurs when mutual defection (conflict) occurs as a dominant strategy of both states. Chicken Game corresponds to cooperation or defection based on states' belief systems and preferences (Osborne & Rubinstein, 1994). Stag Hunt leads to cooperation as the optimal strategy for both states (Skyrms, 2004). The resultant outcomes are again analysed by using various methods like Nash Equilibrium (representing a situation where neither state can unilaterally improve its payoff by changing strategy) and Pareto Optimality, representing a situation in which no state can increase its payoff without affecting the other state (Ingham, 2024; Brisset & Gillon, 2015).

Keeping the complexity of decision-making in international relations in view, there is a dire need to select an appropriate model cautiously, as it also involves the motivational level and other dynamics of states (players) in the given context. Following is a hypothetical mathematical formulation of the game theoretical model based on a specific assumption and game structure to determine whether game theory applies to real-world scenarios. It is an extension of the model discussed above with extended parameters in a more formalised and concise manner, facilitating in-depth analysis and problem-solving.

Table 2. Mathematical Formulation of the Game Theoretical Model

Assumption	In the absence of a centralised authority, there is a strategic interplay between two States (P1 and P2), having a contentious association. Both States have distinctive strategies (S1 and S2) to cooperate (C) or defect (D). Their Payoffs (linked to each strategy) are characterised by utility functions (U1 and U2), which highlight the concerns and preferences of each State. U_1 {S_1, S_2} and U_2 {S_1, S_2} will be the payoffs of State 1 and State 2, respectively.
Game	In this game between two States (P1 and P2), where they do not trust each other, both have a choice (S1, S2) to make either cooperate (C) or defect (D) to achieve the desired outcome (U1, U2). U1 and U2 are the number of points that states (P1 and P2) get, respectively, for each combination of their choices. Therefore, G=<P1, P2>, {S1, S2}, {U1, U2}> represents a strategic contest.
Equations/Analysis	• **Nash Equilibrium:** A situation in which no State can improve its payoff (utility) by unilaterally changing its strategy, assuming that the other State will keep its strategy unchanged. In that case, the Nash equilibrium can be identified by using the equations $\partial U1/\partial S1 = 0$ and $\partial U2/\partial S2 = 0$. Here, $\partial U1/\partial S1 = 0$ represents that there is no change in P1's payoff by changing its strategy, and $\partial U2/\partial S2 = 0$ represents no change in P2's payoff by changing its strategy. • **Pareto Optimality:** Pareto Optimality, in a situation where no State can improve its payoff without making other State suffer. Mathematically, it can be represented as $\partial U1/\partial S1 + \partial U2/\partial S2 = 0$ (any improvement for one State comes at the expense of the other). • **Security Dilemma:** In a hypothetical situation where cooperation leads to a lesser outcome than defection for both States or vice versa. It is mathematically represented as $U1(C,C) < U1(D,D)$ and $U2(C,C) < U2(D,D)$, where $U1(C,C) < U1(D,D)$ represents that P1 gets a lower payoff by cooperating when P2 cooperates and $U2(C,C) < U2(D,D)$ represents that P2 gets a lower payoff by cooperating when P1 cooperates. In a Security Dilemma, the payoff for both States defecting (D,D) is often more significant than the payoff for both States cooperating (C,C), but less than the payoff for one State defecting and the other State cooperating (D,C or C,D).

Source: Author

Keeping Table 2 in consideration, real-world examples related to the model are available. During the Cold War, the United States and the Soviet Union (P1 and P2) were engaged in a strategic interplay regarding nuclear arsenals (i.e., an arms race). They had a choice to engage in cooperation (C) in order to reduce the number of their nuclear weapons or opt for defection (D) by augmenting their military capabilities (weapon stockpiles). The payoffs associated with each strategy were influenced by mutually assured destruction (MAD), where cooperation led to heightened security and defection would increase vulnerabilities. When both states defect (D) rather than cooperate (C), there is a possibility of a security dilemma, aside from the fact that cooperation would yield positive outcomes for both. Security Dilemma often arises from the apprehension of being exploited or targeted by the other state, eventually culminating in suboptimal results for both states. In the case of the Cuban Missile Crisis, a Nash equilibrium appeared when both states, the United States and the Soviet Union, agreed to withdraw their missiles because neither state could improve the outcome by independently changing its strategy, assuming that the other state's strategy remained unchanged. However, the Nash equilibrium does not need to guarantee Pareto optimality. It indicates that there are alternative strategies that yield better results. For example, the Paris Agreement on Climate Change opted for cooperation (C) to reduce Greenhouse Gas Emissions. It leads to a Pareto optimal outcome because reducing Greenhouse Gas Emissions would benefit all states.

The Game theory model provides insight into the hitches faced by states while opting for cooperation in an anarchic system because states often struggle to balance their security concerns and potential advantages. It also highlights the significance of credibility, trust and communication for states involved in strategic interaction, as they have to make credible commitments necessary to achieve cooperative outcomes. Drawing realist and liberalist conclusions out of the model, it emphasises that there is a need to balance out states' interests and collective gains.

Network Analysis and International Relations

Network Analysis is a set of techniques derived from network theory, which has evolved from Computer Science to demonstrate the power of social network influences (Smiraglia, 2015). These techniques are used in various interdisciplinary fields, including international relations, to study the interactions or relationships among a diverse range of actors (or states) within a network or system, such as diplomatic interactions, economic relationships, security alliances, transnational networks, etc. In network analysis, nodes represent states; edges denote the connections between states, and potential outcomes incorporate the network's structure, unveiling clusters, centrality measures and other characteristics for understanding conflict and cooperation. Centrality measures calculate the degree, closeness, betweenness and eigenvector centrality to categorise influential states in a network. Clusters of states with shared characteristics are distinguished by using community detection algorithms in order to study different patterns of conflict or cooperation. Identifying the shortest paths between states helps to analyse chances for mediation or arbitration (Hafner-Burton et al., 2006; Hoffman & Lebovic, 2012; Martin & Harnisch, 2010).

The network analysis approach identifies potential strategic points of influence to foster cooperation or mitigate conflicts. In a real-world example, the European Union (EU) demonstrates the cooperation among European states that share characteristics and behaviours towards EU institutions in pursuit of common objectives, economic integration, and security collaboration. Another example is the United States and China, as they have pivotal positions in the international system. Both have elevated centrality scores and claim multifaceted relationships in diversified spheres, such as diplomacy, trade, and security. There are various techniques for network analysis, such as Graph theory (Carlson, 2024), Network Visualisation (e.g., Gephi, NetworkX), Centrality Measures (Peng et al., 2018), Community Detection Algorithms (Mao et al., 2017), Shortest Path Algorithms (Gries, 2017) and Network Metrics. Game theory can be incorporated into the Network Analysis model to better understand conflict and cooperation dynamics in an unstructured system.

Table 3. Mathematical Representation of Network Analysis Model

Assumption	While using Graph theory, let us assume that G=(V, E) is a graph that represents a network of States in which States interact and influence each other. In this equation, V is a set of nodes (States), labelled from 1 to n, and E is a set of edges (relationships) between nodes (States), represented by pairs of nodes. There are variables, such as adjacency matrix (A), distance (D), centrality (C), and community partition (P), that collectively capture the connectivity, distances, importance, and clustering of states in the network.
Game Structure	• G = (V, E) is a graph representing a network of States. • V = {1, 2, …, n} is a set of nodes (states). • E = {(i, j) | i, j ∈ V} is a set of edges (relationships) between nodes (States). • A = [a_ij] is an Adjacency Matrix representing the edges (relationship) between nodes (States). If there is an edge between nodes i and j (i.e., they have a relationship), then a_ij = 1; otherwise, a_ij = 0. • D = [d_ij] is a Distance Matrix representing the shortest paths between nodes (States). In this case, d_ij represents the shortest path distance between nodes (States) i and j. • C = [c_ij] is a Centrality Matrix representing the importance of each node (State). In this case, c_ij measures the importance of node (State) i in the graph, considering various centrality metrics (e.g., degree, closeness, and betweenness). • P = {P_1, P_2, …, P_n} is a set of partitions (communities) in the graph, where each P_i represents a group of nodes (States) that are densely connected within the community.
Equations	• **Graph Structure:** G = (V, E) • **Adjacency Matrix (A)** can be used to calculate various network metrics, such as: o density(G) = |E| / (|V| * (|V| - 1)) o clustering_coefficient(G) = (|E| / (|V| * (|V| - 1))) * (|V| / (|E| + |V|)) • **Distance Matrix (D)** can be used to calculate the shortest paths between nodes, such as d_ij = min{k | (i, k)(k, j) ∈ E, k ∈ V} • **Centrality Matrix (C)** can be used to calculate various centrality measures, such as: i. degree_centrality(i) = |{j | (i, j) ∈ E}| ii. closeness_centrality(i) = 1 / sum{d_ij | j ∈ V, j ≠ i} iii. betweenness_centrality(i) = sum{k,l ∈ V, i ≠ k ≠ l} (d_kl - d_kj - d_jk) iv. eigenvector_centrality(i) = max{λ | Ax = λx} • **Partition Matrix (P)** can be used to identify communities in the graph, such as P_i = {j | (i, j) ∈ E, j ∈ V}.
Analysis:	This model helps to find connectivity and important relationships between States (edges), shortest paths and distances between States, importance and centrality of each State, and community structure and clustering within the network. After getting detailed information, the model enables a deeper understanding of global politics and diplomacy.

Source: Author

Keeping the network analysis approach in view, the international system can be conceived as a network of sovereign states whose structure is intertwined with diplomatic, economic, and security relationships. In the real world, the EU represents a cluster of states with dense diplomatic and economic ties. Also, the Association of Southeast Asian Nations (ASEAN) represents a network of states with shared regional interests, but it is not as densely tied as the EU. The adjacency matrix indicates the presence or absence of diplomatic relations, trade agreements, and security alliances between the states. In the contemporary world scenario, the United States is maintaining a dense network of economic and diplomatic relationships with European states. Similarly, China has a robust network of trade agreements with Asian states. While defining the most effective path for cooperation or conflict, the distance matrix helps to understand the relationship between the United States and China, thus indicating that robust diplomatic channels would be the most advantageous path in shaping their relationship. The centrality measures reveal the influence and significance of the United States and China. The United States, owing to its wide-ranging diplomatic and military presence, attains a high level of centrality in the world, whereas China's centrality scores on its increasing economic and political influence. Regarding community detection algorithms, BRICS shows a community of emerging economies with shared development goals. Network analysis can also help to understand the Russia-Ukraine conflict, amidst the role of influential states in the system.

Carrying forward the debate between realists and liberalists, the network analysis approach yields distinct outcomes. As has already been discussed, realists underline states' self-interest and security concerns while highlighting hegemonic power or potential threats to stability. The same can be determined through centrality measures *vis-à-vis* community detection algorithms, to help understand the role of alliances. On the other hand, liberalists value cooperation and shared interests among states, which a dense network of relationships and clustering patterns can indicate. States with high centrality scores act as leaders or architects of international cooperation. However, the role of international institutions and norms in shaping international relations can be identified through network structure and community detection. Both perspectives can have insights from network analysis.

Dynamical Systems and International Relations

A dynamical system approach is used to analyse evolving and complex systems while examining system dynamics, including behaviour, patterns, and trajectories. In international relations, it offers an alternative to traditional qualitative and quantitative methods by providing a dynamic and endogenous point of view, which allows for understanding the dynamic interactions between variables and making short- and long-term projections for alternative policy choices (Fisunoglu, 2018). This approach is used to understand the dynamics of conflict (modelling the escalation and de-escalation of conflicts, state interactions and the consequences of interventions), international cooperation (analysing the emergence and sustainability of cooperation mechanisms, including the roles of incentives, norms and institutions), global governance (exploring the dynamics of international institutions, systems and networks, inclusive of their adaptability and resilience), political regimes (assessing the stability and instability of political systems and impact of economic, social and political factors), and international political economy (modelling the dynamics of trade, investment and financial flows, factoring in the consequences of policy interventions and external shocks).

The key concepts in dynamical systems as applied to international relations consist of phase transitions (abrupt shifts in system behaviour, such as initiation of conflict or emergence of cooperation), attractors (stable states or patterns towards which the system gravitates, such as a stable peace or recurring conflict), bifurcations (alterations in system behaviour due to minor adjustments in parameters or initial conditions), feedback loops (cycles of cause and effect that can stabilise or destabilise the system) and nonlinearity (the occurrence where minor changes have notable and disproportionate effects). Researchers employ various computational tools, such as differential equations, agent-based modelling, network analysis, system dynamics modelling and machine learning algorithms (Ward, 2019; Weidmann, 2017). An illustrative example of modelling conflict and cooperation by using a dynamical system approach is given below.

Table 4. Mathematical Representation of a Dynamical System Model

Assumption	In an anarchic system, there is an arms race between two sovereign States (A and B). Each State's military spending is influenced by the other State's military expenditures.
Game Structure	It is a differential game in which the rate of change in military spending fluctuates over time.
Equation	• $dx/dt = \alpha y - \beta x$ (rate of change in State A's military spending) • $dy/dt = \gamma x - \delta y$ (rate of change in State B's military spending) Note: The rate of change in military spending (dx/dt) is proportional to the opponent's military spending (y) and inversely proportional to one's own military spending (x). • α represents the degree to which State A feels threatened or responsive to State B's military buildup. • γ represents the degree to which State B feels threatened or responsive to State A's military buildup. • β and δ represent the cost or burden of maintaining a high level of military spending.
Variables	• x: State A's Military Spending • y: State B's Military Spending • α, β, γ, δ are constants representing States' response coefficients • t: time • dx/dt and dy/dt: Rates of change of military spending over time
Analysis	Stability of the equilibrium points Potential for escalation or disarmament

Source: Author

This model highlights the dynamic interaction between two states' military spending, in which each state's military buildup is influenced by the other state's military expenditure as well as its own military spending. This phenomenon in international relations is usually referred to as an arms race. Alternatively, it is known as stability in military spending, depending upon the values of intervening factors. A scenario in which each state tries to outspend the other is referred to as a security dilemma, where one state's military buildup leads to insecurity and military buildup in the other state, and stability in military spending, where both states reach a mutually acceptable level of military expenditure. A crude example from the real world is the nuclear arms race between the United States and the Soviet Union during the Cold War period. Researchers, while using historical data, can estimate parameters and stimulate dynamics to analyse how this arms race escalated and eventually stabilised (Pilch, 2001).

Keeping the dynamics and parameters of the given model (Table 4) in consideration, there are various scenarios in international relations where a dynamical system approach can be used, e.g., the formation of a mutually beneficial alliance (where both states cooperate and receive benefits), the development of a trade agreement (where both states reduce tariffs and increase trade), and the establishment of a peace treaty (where both states agree to end hostilities and cooperate on security issues). In the case of cooperation between two states, incentives or bonuses can characterise economic benefits, security guarantees, diplomatic recognition, and international prestige. Similarly, states' decisions to cooperate are influenced by political ideology, national interests, public opinion, and leadership preferences. A real-world example is the formation of the EU as a dynamic cooperation process. Using a dynamic system approach, researchers can study the EU's expansion and analyse how the cooperation levels of various states increased over time (Volgy & Others, 2012).

Dynamical systems can also study the impact of external shocks, such as economic crises, political unrest, alliance formation, and natural disasters, on international cooperation and conflict, involving a system of differential equations. Various techniques, including bifurcation analysis, chaos theory, network theory, agent-based modelling and system dynamics, are applied to understand how external shocks influence states' relationships. These methods help identify tipping points, regime changes, and the unpredictable nature of international relations under external shocks.

Case Study: Mathematical Modelling of Russia-Ukraine Conflict

The Russia-Ukraine armed conflict, which originated in 2014, was sparked by the ousting of Ukrainian President Viktor Yanukovych, who was pro-Russia, in a popular uprising. It led to Russia's annexation of Crimea and sparked a separatist movement in Eastern Ukraine. The recent conflict is considered an extension of previous hostilities. Up till now, the conflict has resulted in a substantial death toll and displacement of a huge Ukrainian population, besides enormous economic and infrastructure loss. This ongoing armed conflict can be examined through mathematical modelling across various dimensions. For example, conflict dynamics can be analysed by using differential equations or agent-based models to simulate escalation and de-escalation of the conflict. Game theory or network analysis can be employed to understand tactics and strategies used by both states in military operations. Game theory or decision analysis helps model political negotiations and the probability of reaching a peace agreement. Econometric models can help analyse the economic impact, including sanctions and trade disruption. There are several variables available that can be used to model different facets of the conflict. Military variables include the number of troops, deployment, use of sophisticated weaponry, military equipment, and military strategies and tactics. Political variables include diplomatic efforts and negotiations, political leadership and will, and public opinion and support. Economic variables are sanctions, trade disruption, GDP, inflation, and currency depreciation. Social variables include civilian casualties, humanitarian assistance, media, and propaganda. Besides, there are geographical, time, international, and uncertainty variables. However, the selection of variables directly correlates to the research questions and the modelling approach used.

The complexity of the Russia-Ukraine conflict holds the importance of an integrated approach employing game theory using utility functions and payoff matrices to examine strategic interactions among Russia, Ukraine and other international stakeholders, predicting the outcome; Dynamical system by developing stock and flow diagram to understand conflict dynamics and its impact, apprehending the feedback loops and nonlinear effects of diplomatic efforts, public opinion and economic sanctions; and network analysis by assigning nodes and edges weights and computing network centrality measures to comprehend inter-relationship between entities involved, structural properties of conflict network, identifying key actors and potential vulnerabilities. It is a kind of hybrid approach permitting policymakers to stimulate different scenarios and predict outcomes of various diplomatic, economic and military strategies employed in the conflict.

In the context of modelling using game theory, the simplest scenario could be a strategic interplay between Russia and Ukraine, where both states need to choose either negotiation (diplomatic efforts to resolve the conflict) or escalation (through military means) to illustrate outcomes through a payoff matrix based on their strategic choices. If both states opt for negotiation (N, N), they both receive the same utility value, highlighting resolution and a degree of cooperation. Suppose Ukraine chooses negotiation and Russia opts for escalation; in that case, there is a possibility that Ukraine will receive less utility value (indicating loss) and Russia will gain maximum utility value (indicating gain), and vice versa. If both states go for escalation (E, E), then the probability of escalation (p_E) is high, denoting that the conflict will escalate into a full-scale war. In this case, the model represents a security dilemma in which both states are stuck in a cycle of escalation due to competing interests and mistrust.

Using the system dynamics approach for modelling the Russia-Ukraine conflict as a complex system, where the behaviour of one side effects the behaviour of the other, the stocks and flows signify that the military strength of Russia and Ukraine (RMS and UMS) can be increased through military buildup efforts (RMBR and UMBR), economic resources (RER and UER) can be affected by economic aid (UEAI) or sanctions (RESI). International diplomatic pressure (IDP) can be increased through diplomatic efforts (DERC). This analysis also refers to a security dilemma, as military buildups by both states mutually intensify fears, and conflict escalates further. It would also entail economic ramifications, including a substantial increase in military expenditures and depletion of economic resources (which could be covered up through aid or sanctions). International interference or diplomatic endeavours could exert pressure on both states to seek a resolution to the conflict.

When conducting a network analysis of the Russia-Ukraine conflict, the mathematical modelling would entail the representation of conflict as a network, a complex web of relationships between states, international organisations, and institutions. Each node (actor) has its own attributes, such as political leaning, military strength, and economic resources, which influence its behaviour and interactions with other nodes. The edges (relationships) between nodes represent diplomatic, economic, military, and institutional ties, which can be strong or weak, cooperative or conflictual. The weight of each edge indicates the strength of the relationship, and the type of edge (diplomatic, economic, military) indicates the nature of the interaction. The model also helps to identify central nodes (e.g., Russia, Ukraine, US, EU) that play a crucial role in shaping the conflict, analysing the cohesion of the network, revealing the strength of alliances and intensity of conflicts, simulating scenarios, such as diplomatic efforts to resolve the conflict or economic sanctions imposed on Russia and predict the likelihood of conflict escalation or resolution based on network structure and node attributes. The model represents complex interdependence, highlighting how their relationships shape conflict dynamics.

The Russia-Ukraine conflict is a multilayered challenge necessitating an all-inclusive approach to its analysis and finding solutions. In that case, realists and idealists need a balanced approach that should take into account national interests and efforts for cooperation. In this regard, realists need to advocate for the maintenance of military deterrence to pursue national security *vis-à-vis* readiness to adopt abrupt shifts in conflict dynamics. At the same time, idealists are responsible for promoting diplomatic efforts and pushing for a sustainable negotiating process, international cooperation, and economic interconnectedness.

Conclusion

Mathematical modelling in the contemporary context of conflict and cooperation offers innumerable prospects for diversified theoretical approaches in international relations so that they can converge on a single agenda point for workable conflict resolution. Although mathematical modelling pushes for military power and strategic interests in an anarchic setting, the need for diplomatic efforts and international cooperation are not ruled out to maintain peace and stability. As it has been observed from the preceding discussion that, employing mathematical tools in analysing real-world scenarios helps provide insight into the complex dynamics of the international system, it is, therefore, imperative to adopt integrated approaches to address underlying issues driving conflicts and exploiting chances for cooperation. For instance, while addressing the issues related to terrorism, the dynamical system approach assists in understanding the propagation of terrorist ideologies, whereas network analysis identifies critical actors within terrorist networks. Similarly, exploring cybersecurity involves network analysis to investigate cyberattacks, and Game theory explores the dynamics of cyber conflict. Mathematical modelling is equally valuable for studying environmental politics, the impact of climate change, and the efficacy of international agreements. Moreover, migration flow dynamics and the influence of refugee policies can also be comprehended through modelling. These illustrations accentuate mathematical modelling in addressing crucial global issues within international relations. They provide valuable insights and foster a deep understanding of the complex dynamics that shape our world today.

However, using mathematical modelling in international relations also has its limitations and challenges. It is difficult for a mathematical model to fully capture complex human behaviour, particularly in diplomacy and state interactions. Moreover, the impulsive nature of international relations can make modelling difficult because it is hard to create reliable models accounting for all relevant variables. Mathematical models undertake simple assumptions and manageable data, thus putting a risk of oversimplification. It may lead to omitting important factors influencing states' behaviours and outcomes. Another related challenge is the availability of accurate and quality data regarding a wide range of variables, as most of the information is kept classified or declared sensitive and not accessible to researchers. Additionally, including and weighing the importance of variables can involve an element of prejudice, which could put the model at risk and impact its accuracy and reliability. Lastly, overreliance on the modelling approach may displace human judgement and qualitative analysis and limit comprehension of the complexities involved in states' cooperation and conflict.

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